Optical interferometry – a gentle introduction

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Motivation

- A – Uninterested: here for the holiday.
- B – Might be interested but sceptical: prove it to me!
- C – Possibly interested: need to learn more.
- D – Interested: want to understand how I can use this.
- E – I know this is exciting: looking to raise my game.

Outline

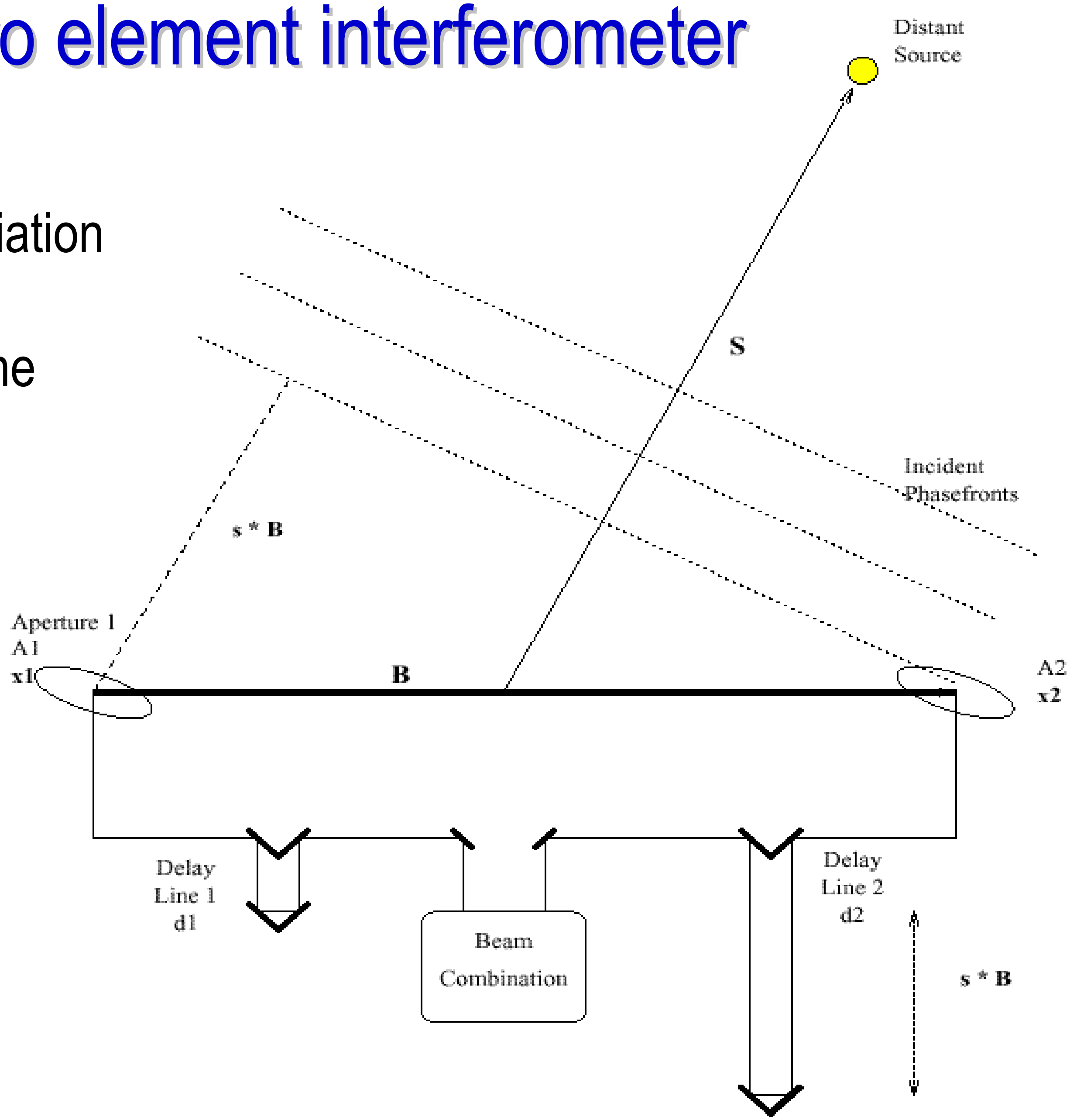
- Preamble
- A simple interferometer
- Fringe parameters
- Information about the source
- Typical visibility functions
- Putting it all together
- Summary

Preamble

- Learning interferometry is like learning to surf:
  - You have to want to do it.
  - You start in the shallows.
  - Having an expensive surf-board doesn't help.
  - You don't have to know how to make surf-boards.
  - Knowing how to surf won't help you escape a charging tiger.
- This is a school:
  - I will assume nothing.
  - You should assume nothing – don't guess.
  - Ask questions.
  - If you don't understand ask the teachers.
- I am not trying to sell you a surf board.

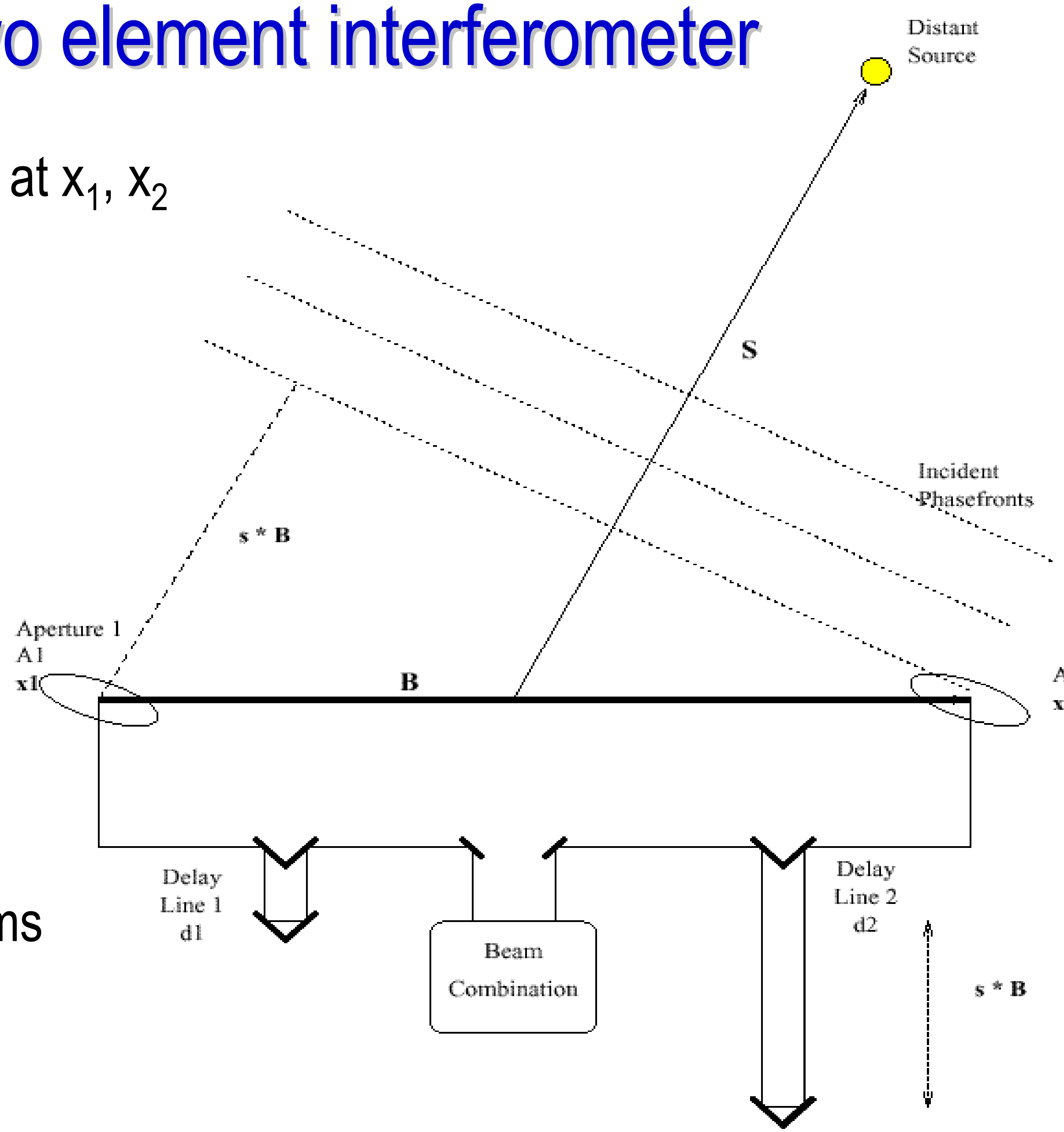
A two element interferometer

- Sampling of the radiation
- Compensation for the geometric delay
- Combination of the beams
- Detection of the resulting output



A two element interferometer

- Telescopes located at x_1, x_2
- Baseline $B = (x_1 - x_2)$
- Pointing direction is S
- Geometric delay is $\hat{s}.B$, where $\hat{s} = S/|S|$
- Paths along two arms are d_1 and d_2



The output of a 2-element interferometer (i)

- The fields at two apertures can be written as:
  - $\psi_1 = A \exp(ik[\hat{s}.B + d_1]) \exp(-i\omega t)$ and $\psi_2 = A \exp(ik[d_2]) \exp(-i\omega t)$
- So the sum of the fields is:

$$\Psi = \psi_1 + \psi_2 = A \left[\exp(ik[\hat{s}.B + d_1]) + \exp(ik[d_2])\right] \exp(-i\omega t)$$

- And hence the time averaged intensity, $\langle \Psi\Psi^* \rangle$, will be given by:

$$\langle \Psi\Psi^* \rangle \propto \langle\, [\exp(ik[\hat{s}.B + d_1]) + \exp(ik[d_2])] \times [\exp(-ik[\hat{s}.B + d_1]) + \exp(-ik[d_2])]\, \rangle$$

$$\propto 2 + 2\cos(\, k\, [\hat{s}.B + d_1 - d_2]\,)$$

$$\propto 2 + 2\cos(kD)$$

Note, here $D = [\hat{s}.B + d_1 - d_2]$.

This is a function of the path lengths, d_1 and d_2, the pointing direction and the baseline.

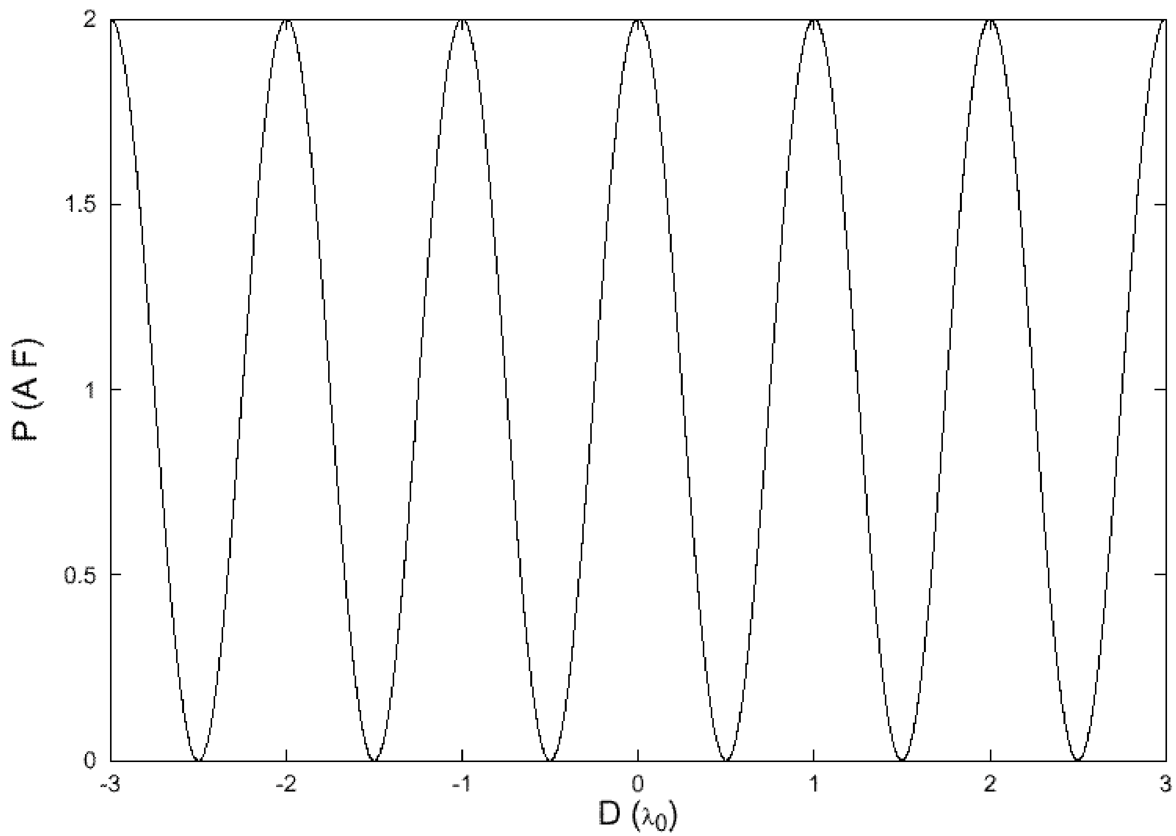
The output of a 2-element interferometer (ii)

Detected power, $P = \langle \Psi \Psi^* \rangle \propto 2 + 2 \cos (k [\hat{s}.B + d_1 - d_2])$

$\propto 2 + 2 \cos (kD)$, where $D = [\hat{s}.B + d_1 - d_2]$

– The output varies co-sinusiodally with D.

– Adjacent fringe peaks are separated by $\Delta d_{1 \text{ or } 2} = \lambda$ or $\Delta \hat{s} = \lambda / B$.

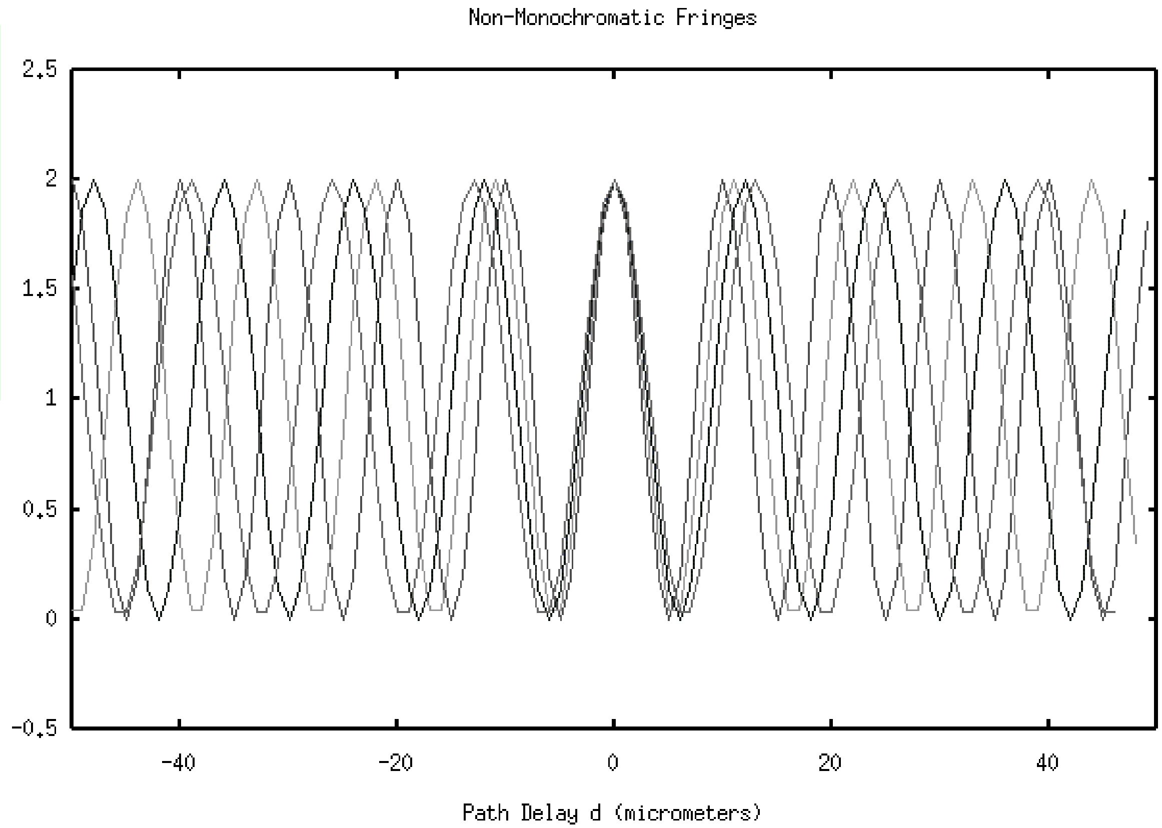


Extension to polychromatic light

- Simply integrate the previous result over a range of wavelengths:
  - E.g for a uniform bandpass of $\lambda_0 \pm \Delta\lambda/2$ (i.e. $\nu_0 \pm \Delta\nu/2$) we obtain

$$P \propto \int_{\lambda_0-\Delta\lambda/2}^{\lambda_0+\Delta\lambda/2} [2+2\cos(kD)]\, d\lambda$$

$$\propto \int_{\nu_0-\Delta\nu/2}^{\nu_0+\Delta\nu/2} 2[1+\cos(2\pi\nu D/c)]\, d\nu$$



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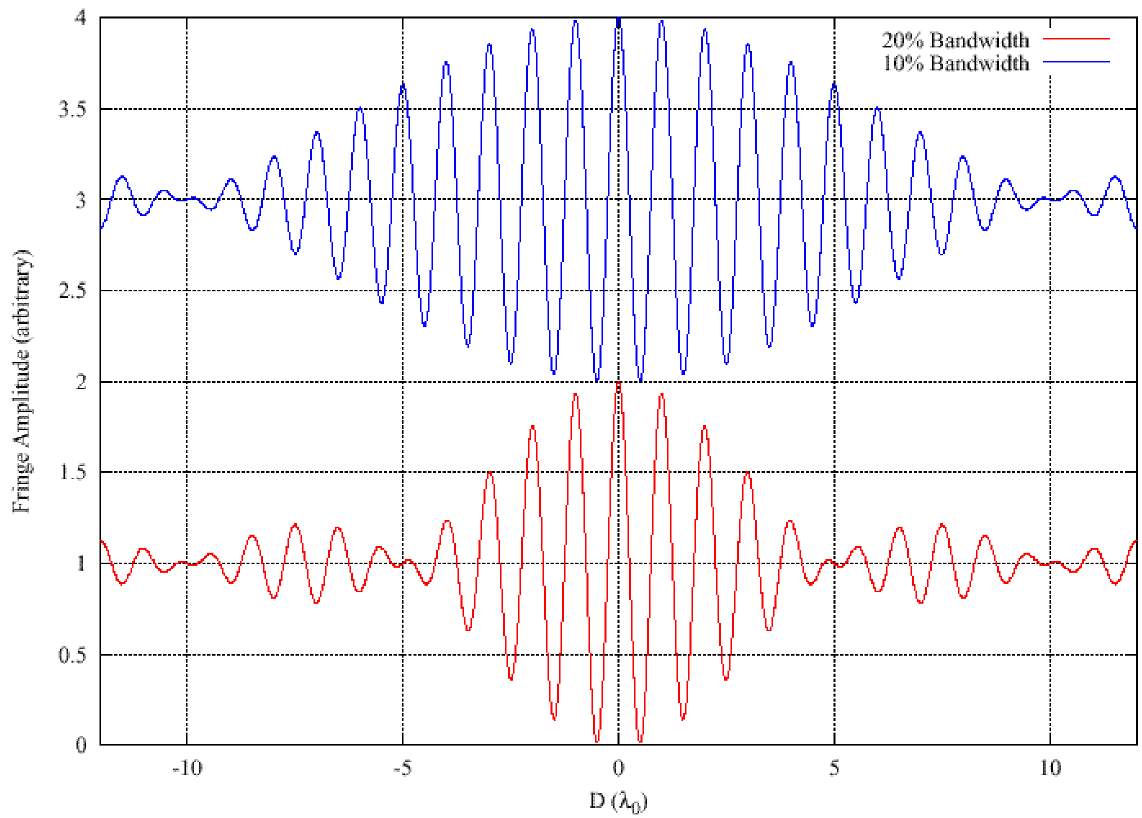
$$= \int_{\nu_0-\Delta\nu/2}^{\nu_0+\Delta\nu/2} 2\,[1+\cos(2\pi\nu\, D/c)]\,d\nu$$

$$= \Delta\lambda\left[1+\frac{\sin \pi D\Delta\lambda/\lambda^2{}_0}{\pi D\Delta\lambda/\lambda^2{}_0}\cos k_0 D\right]$$

$$= \Delta\lambda\left[1+\frac{\sin \pi D/\Lambda_{coh}}{\pi D/\Lambda_{coh}}\cos k_0 D\right]$$



  - NB The fringes are modulated with an envelope with a characteristic width equal to the coherence length, $\Lambda_{coh} = \lambda^2{}_0/\Delta\lambda$.

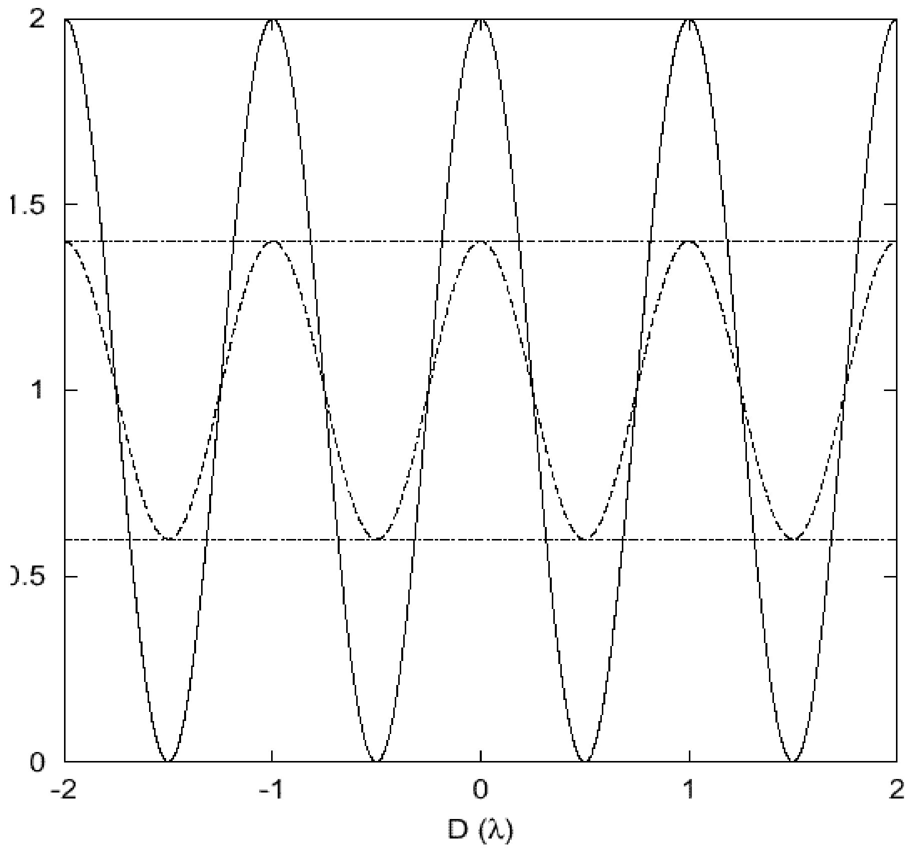
Fringe parameters of interest

- From an interferometric point of view the key features of any interference fringes are their modulation and their location with respect to some reference point.

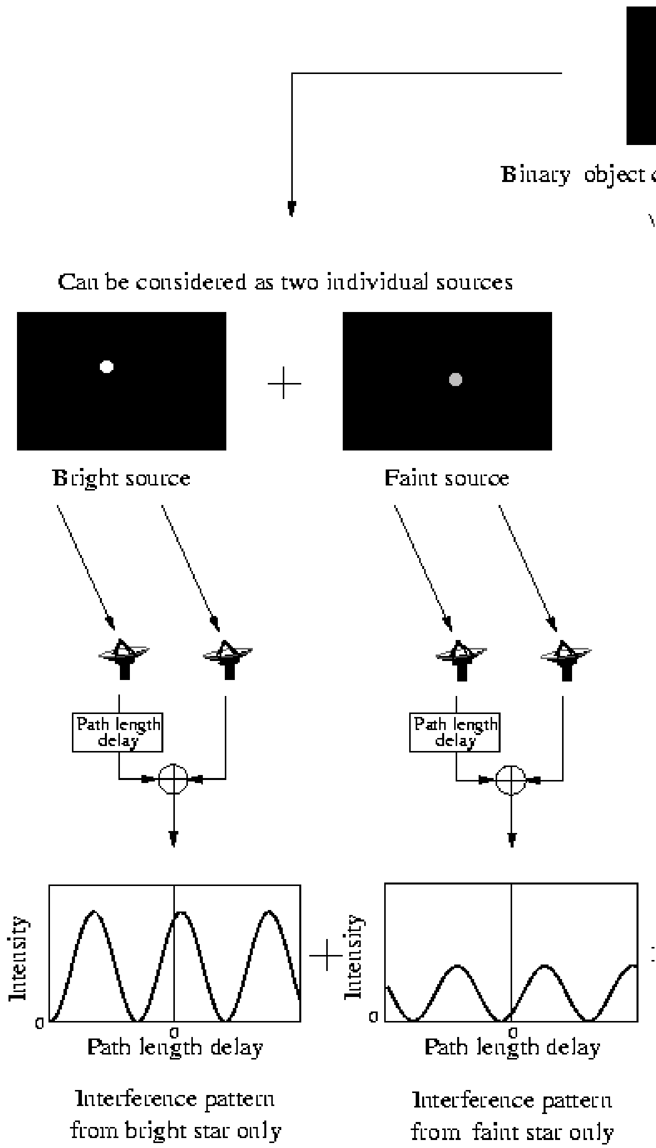
- In particular we can identify:
  - The fringe visibility:

$$V = \frac{[I_{max} - I_{min}]}{[I_{max} + I_{min}]}$$

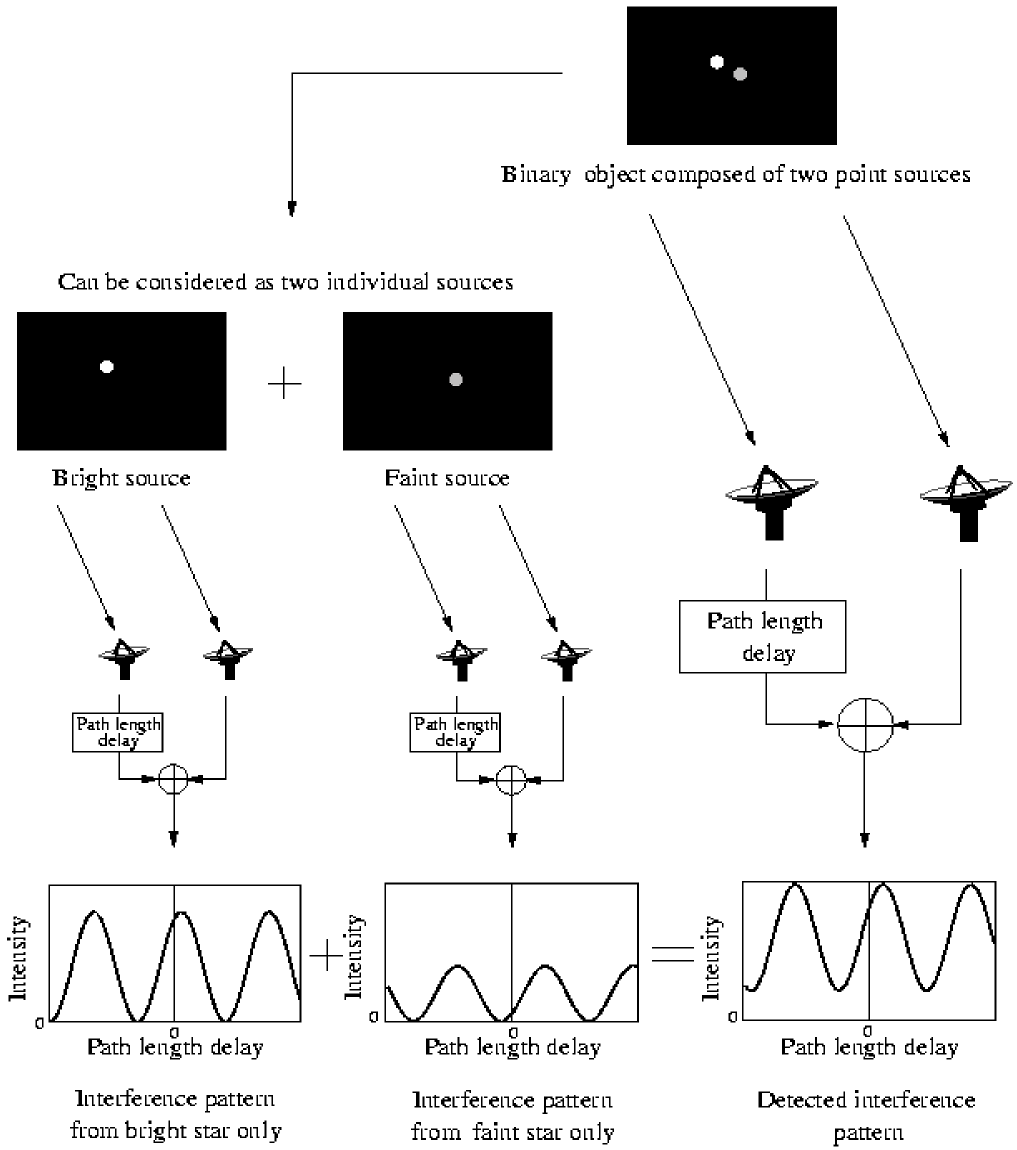
  - The fringe phase:
    - The location of the white-light fringe as measured from some reference (radians).



The essential physics



The essential physics



- The resulting fringe pattern has a modulation depth that is reduced with respect to that from each source individually.
- The positions of the sources are encoded in the fringe phase.

Response to a distributed source

- Consider looking at an incoherent source whose brightness on the sky is described by $I(\hat{s})$. This can be written as $I(\hat{s}_0+\Delta s)$, where $\hat{s}_0$ is the pointing direction, and Δs is a vector perpendicular to this.
- The detected power will be given by:

$$
\begin{aligned}
P(\hat{s}_0, B) &\propto \int I(\hat{s})\left[1+\cos kD\right] d\Omega \\
&\propto \int I(\hat{s})\left[1+\cos k(\hat{s}.B+d_1-d_2)\right] d\Omega \\
&\propto \int I(\hat{s})\left[1+\cos k([\hat{s}_0+\Delta s].B+d_1-d_2)\right] d\Omega \\
&\propto \int I(\hat{s})\left[1+\cos k(\hat{s}_0.B+\Delta s.B+d_1-d_2)\right] d\Omega \\
&\propto \int I(\Delta s)\left[1+\cos k(\Delta s.B)\right] d\Omega'
\end{aligned}
$$

The van Cittert-Zernike theorem (i)

- Consider now adding a small phase delay, δ, to one arm of the interferometer. The detected power will become:

$$P(\hat{s}_0, B, \delta) \propto \int I(\Delta s)\left[1 + \cos k(\Delta s.B + \delta)\right] d\Omega$$

$$\propto \int I(\Delta s)\, d\Omega + \cos k\delta \cdot \int I(\Delta s) \cos k(\Delta s.B)\, d\Omega$$

$$- \sin k\delta \cdot \int I(\Delta s) \sin k(\Delta s.B)\, d\Omega$$

- Introducing the complex visibility V(*k*,*B*) we can write:

$$V(k, B) = \int I(\Delta s) \exp[-ik\Delta s.B]\, d\Omega$$

so that:

$$P(\hat{s}_0, B, \delta) \propto \int I(\Delta s)\ \ d\Omega + \cos k\delta\, \mathrm{Re}[V] + \sin k\delta\, \mathrm{Im}[V]$$

$$P(\hat{s}_0, B, \delta) \ = \ I_{total} + \mathrm{Re}\left[V \exp[-ik\delta]\right]$$

The van Cittert-Zernike theorem (ii)

- Now assume $\hat{s}_0$ = (0,0,1) and Δs is small and $\approx (\alpha,\beta,0)$, with α and β angles measured in radians.
- This implies $$V(k,B) = \int I(\alpha,\beta)\exp[-ik(\alpha B_x + \beta B_y)]\, d\alpha\, d\beta$$
- So that $$V(u,v) = \int I(\alpha,\beta)\exp[-i2\pi(\alpha u + \beta v)]\, d\alpha\, d\beta$$

where *u* (=B_x/λ) and *v* (=B_y/λ) are spatial frequencies with units rad^{-1}.

- Since $$P(\hat{s}_0, B, \delta) = I_{total} + \mathrm{Re}\left[V \exp[-ik\delta]\right]$$

what this means is that the interferometer response measures the Fourier transform of the sky brightness distribution.

This is the van Cittert-Zernike theorem.

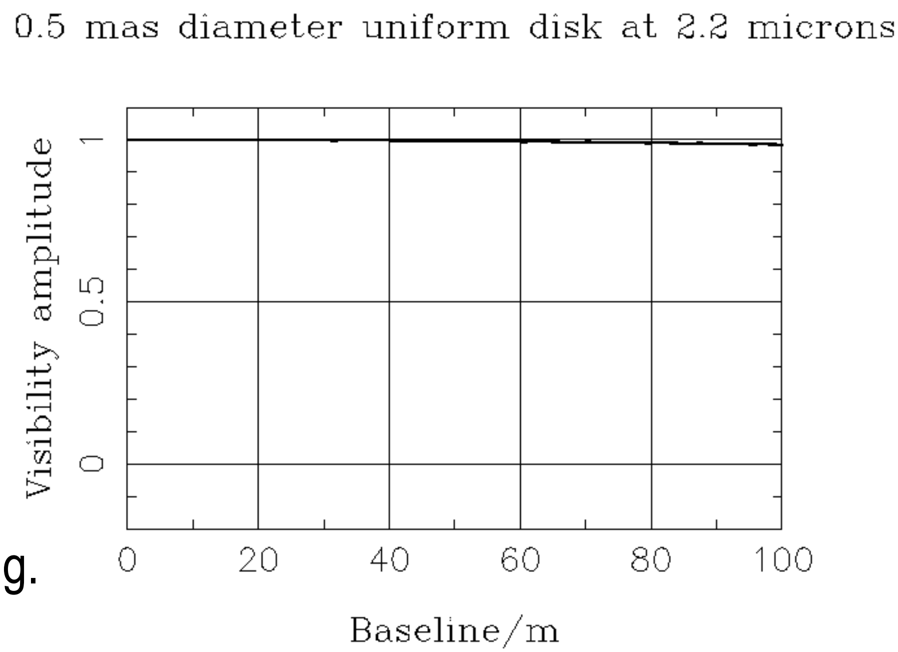
Simple sources (i)

$$V(u) = \int I(l)\, e^{-i2\pi(ul)}\, dl \,/ \int I(l)\, dl$$

- Point source of strength A_1 and located at angle l_1 relative to the optical axis.

$$V(u) = \int A_1\delta(l-l_1)\, e^{-i2\pi(ul)}\, dl \,/ \int A_1\delta(l-l_1) dl$$
$$= e^{-i2\pi(ul_1)}$$

- The visibility amplitude is unity $\forall$ *u.*
- The visibility phase varies linearly with *u* (= B/λ).
- Sources such as this are easy to observe, but of little interest if you've built an interferometer for high-angular resolution imaging.



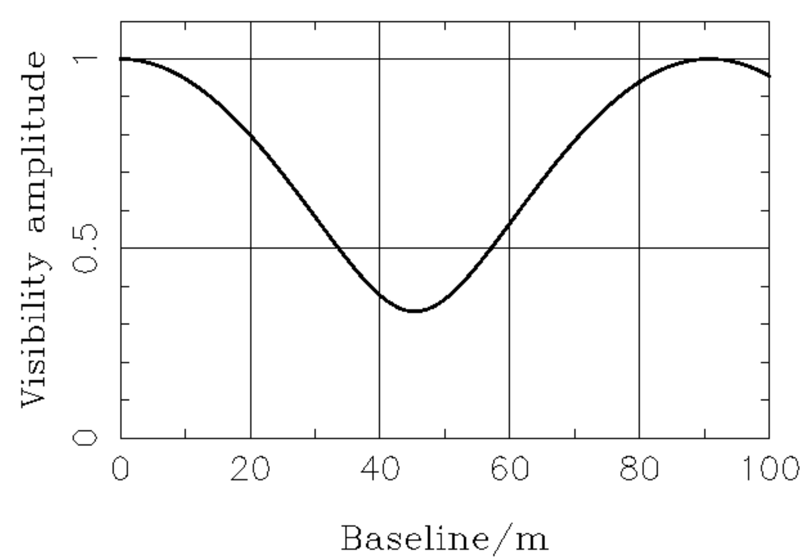
Simple sources (ii)

$$V(u) = \int I(l)\, e^{-i2\pi(ul)}\, dl \,/ \int I(l)\, dl$$

- A double source comprising point sources of strength A_1 and A_2 located at angles 0 and l_2 relative to the optical axis.

$$V(u) = \int [A_1\delta(l) + A_2\delta(l-l_2)]\, e^{-i2\pi(ul)}\, dl \,/ \int [A_1\delta(l) + A_2\delta(l-l_2)]\, dl$$
$$= [A_1 + A_2 e^{-i2\pi(ul_2)}] / [A_1 + A_2]$$

- The visibility amplitude and phase oscillate as functions of *u*.
- To identify this as a binary, baselines from $0 \rightarrow \lambda/l_2$ are required.
- If the ratio of component fluxes is large the modulation of the visibility becomes increasingly difficult to measure.



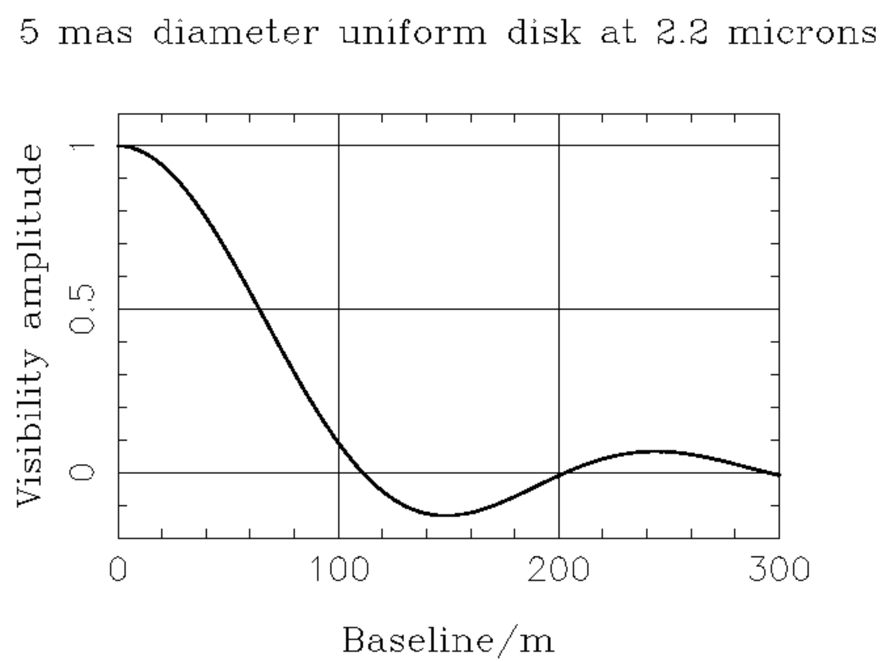
Simple sources (iii)

$$V(u) = \int I(l)\, e^{-i2\pi(ul)}\, dl \,/ \int I(l)\, dl$$

- A uniform on-axis disc source of diameter θ.

$$V(u_r) \propto \int^{\theta/2} \rho\, J_0(2\pi\rho u_r)\, d\rho$$
$$= 2J_1(\pi\theta u_r) / (\pi\theta u_r)$$

- To identify this as a disc requires baselines from $0 \longrightarrow \lambda/\theta$ at least.
- The visibility amplitude falls rapidly as u_r increases.
- Information on scales smaller than the disc diameter correspond to values of u_r where V << 1, and is hence difficult to measure.



Review of interferometric imaging

- The visibility function, $V(u, v) = V(B_x/\lambda,\ B_y/\lambda)$, is the Fourier transform of the source brightness distribution.

- So measure V for as many values of B as possible & perform an inverse FT.

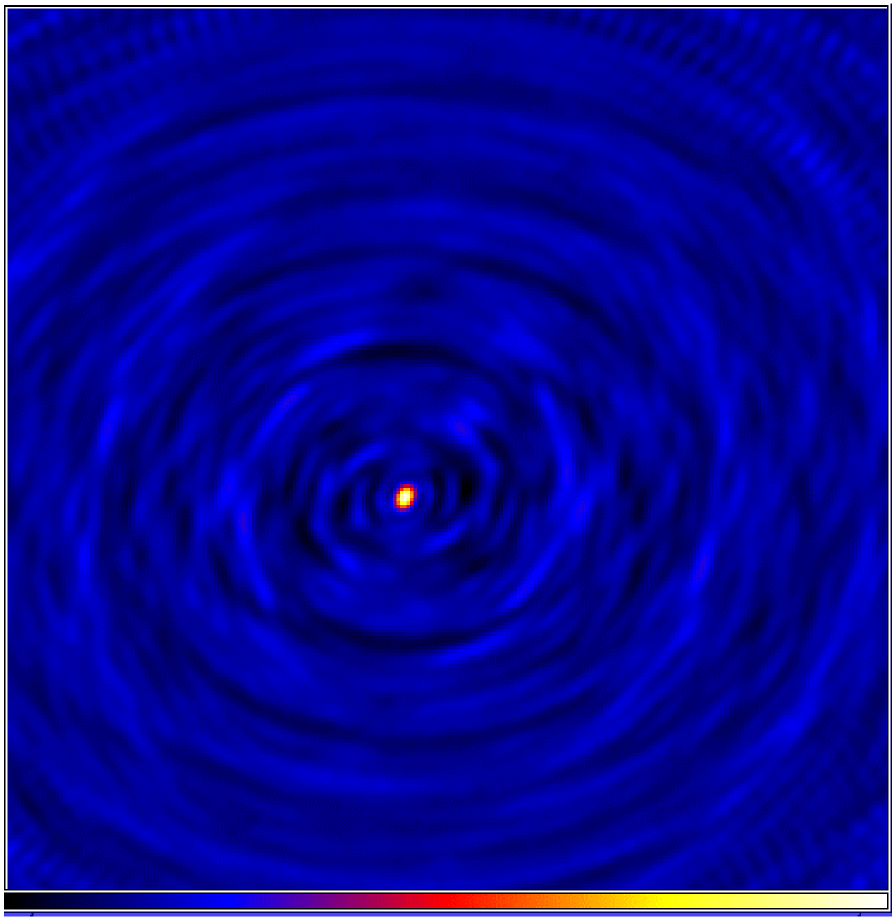
$$I_{norm}(l, m) = \iint V(u, v)\, e^{+i2\pi(ul + vm)}\, du\, dv$$

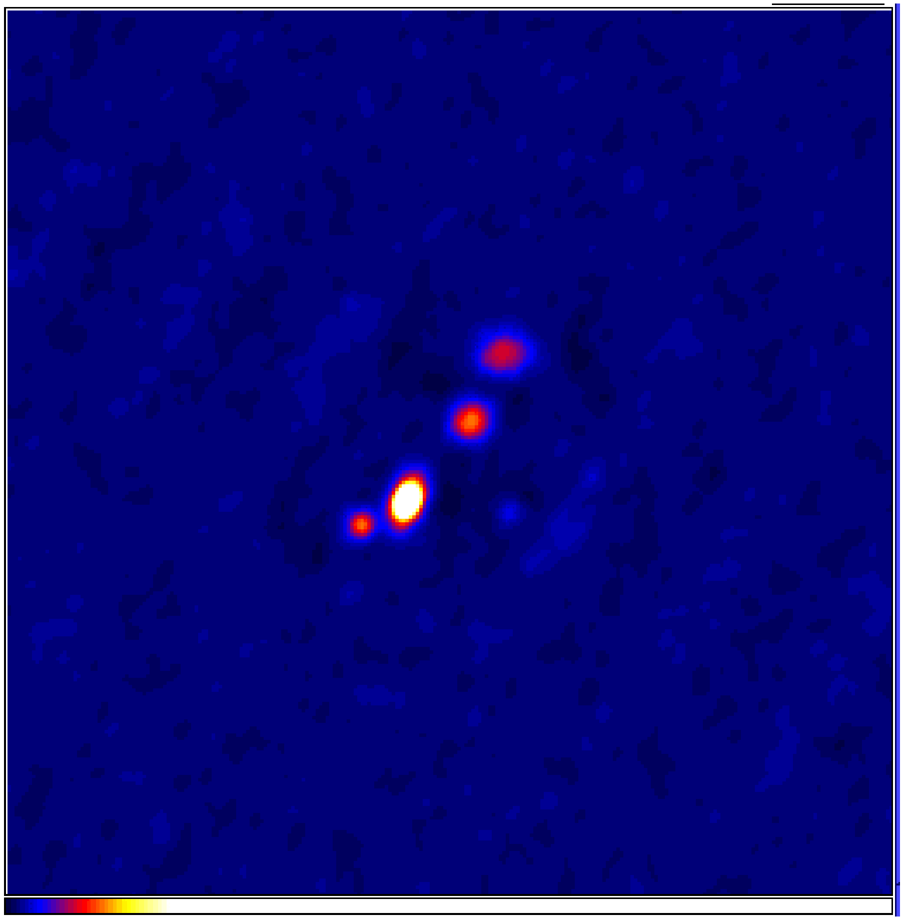
- Since what we measure is a sampled version of $V(u, v)$, what we actually recover is the so-called "dirty map":

$$\begin{aligned} I_{dirty}(l, m) &= \iint S(u, v)\, V(u, v)\, e^{+i2\pi(ul + vm)}\, du\, dv \\ &= B_{dirty}(l, m) * I_{norm}(l, m)\ , \end{aligned}$$

where $B_{dirty}(l,m)$ is the Fourier transform of the sampling distribution, or dirty-beam.

Dirty (and corrected) interferometric images

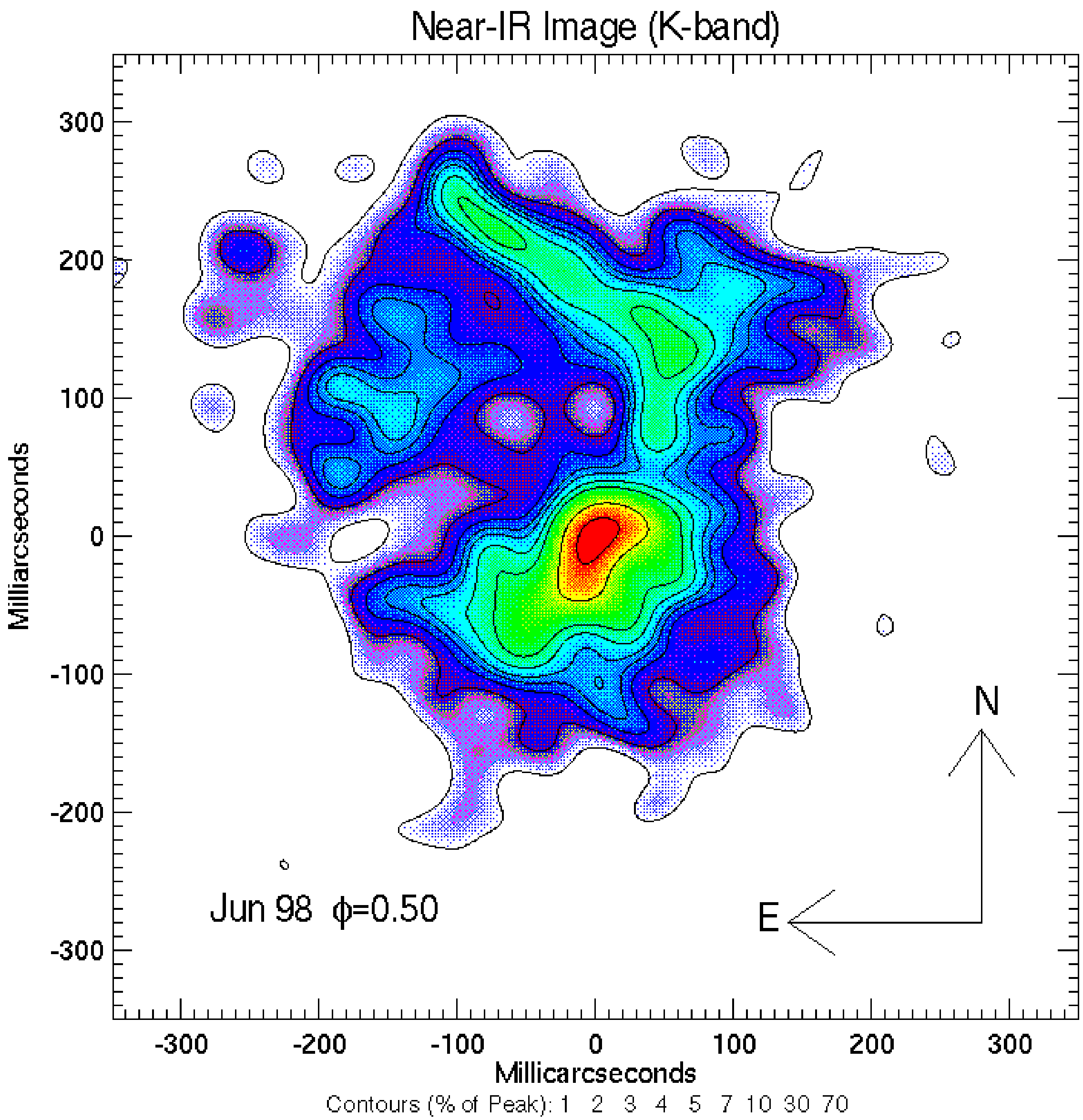




- Correcting an interferometric map for the Fourier plane sampling function is known as deconvolution (CLEAN, MEM, WIPE).

A real astronomical example

K-band image of IRC+10216. Image courtesy of Peter Tuthill and John Monnier.



Summary

- Interferometers measure fringes.
- The fringe modulation and phase are the quantities of interest.
- These measure the amplitude and phase of the FT of the source brightness distribution (the visibility function).
- Any given interferometer baseline responds to a single spatial frequency in the source brightness distribution.
- Multiple baselines are obligatory to build up an image.
- Once many visibility measurements are made, an inverse FT delivers a representation of the source that may (or may not) be useful!